Trigonometry Cheat-Sheet

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Relationships involving sums and differences of angles

$$
\begin{aligned}
\sin(\theta \pm \phi) &= \sin\theta\cos\phi \pm \cos\theta\sin\phi \\
\cos(\theta \pm \phi) &= \cos\theta\cos\phi \mp \sin\theta\sin\phi \\
\tan(\theta \pm \phi) &= \frac{\tan\theta \pm \tan\phi}{1 \mp \tan\theta\tan\phi} \\
\sin 2\theta &= 2\sin\theta\cos\theta \\
\cos 2\theta &= \cos^2\theta - \sin^2\theta \\
\tan 2\theta &= \frac{2\tan\theta}{1 - \tan^2\theta} \\
2\cos^2\theta &= 1 + \cos 2\theta \\
2\sin^2\theta &= 1 - \cos 2\theta \\
2\sin\theta\cos\phi &= \sin(\theta - \phi) + \sin(\theta + \phi) \\
2\cos\theta\cos\phi &= \cos(\theta - \phi) + \cos(\theta + \phi) \\
2\sin\theta\sin\phi &= \cos(\theta - \phi) - \cos(\theta + \phi)
\end{aligned}
$$

The $\tan$ relationships are true providing $\theta + \phi \neq (k + \frac{1}{2})\pi, k \in \mathbb{Z}$.

Vectors

The vector $\mathbf{V} = (x, y, z)$ has length $V \equiv |\mathbf{V}| = \sqrt{x^2 + y^2 + z^2}$.

The scalar product of two vectors $\mathbf{P} = (P_x, P_y, P_z)$ and $\mathbf{Q} = (Q_x, Q_y, Q_z)$ is given by

$$
\begin{aligned}
\mathbf{P}.\mathbf{Q} &= P_xQ_x + P_yQ_y + P_zQ_z \\
&= PQ\cos\theta
\end{aligned}
$$

where P and Q are the lengths of the vectors and θ the angle between them.

The vector product of $\mathbf{P}$ and $\mathbf{Q}$ is

$$
\mathbf{P} \times \mathbf{Q} = \begin{pmatrix} P_yQ_z - P_zQ_y \\ P_zQ_x - P_xQ_z \\ P_xQ_y - P_yQ_x \end{pmatrix}
$$